Correspondence

Linear Pulse-Forming Circuits*

Regarding the above article,[1] I would like to point out that a general solution to this problem, including an error estimation method, was given on pp. 66–71 in the January 15, 1952 Progress Report of the M.I.T. Research Laboratory of Electronics. Further detailed discussion and evaluation of the error criterion was made by Mike Strieby in his paper, "A Fourier Method for Time Domain Synthesis," given at the P.I.B. Symposium on Modern Network Synthesis, April 13–15, 1955.

The method of Gore and Larsen, besides lacking in an error analysis and evaluation method, is restricted to pulse shapes which are symmetrical about a vertical center line and further by the requirement (not stated by Gore and Larsen) that the Fourier representation given by (4), p. 185, shall contain only positive coefficients! The latter restriction, which is necessary in order to insure the realizability of the impedance $Z(p)$, seriously limits the kind of symmetrical pulse shapes obtainable. A detailed discussion of the more general method referred to above, together with other methods of time domain synthesis, will be found in chapter XV of my book on network synthesis, which is scheduled to appear by the middle or end of next summer.

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[1] W. C. Gore and T. Larsen, IRE Trans., vol. CT-3, pp. 182–188; September, 1956.

On the Compensation Theorem*

In a recent note,[1] Papoulis found the coefficients in a perturbation series for the current change in a network branch when a small resistance, R, is introduced in the branch. It is the purpose of this note to give a different expression for the perturbing current.

The method is closely related to that of Papoulis. We expand the perturbed current in powers of R as follows:

$$\begin{aligned} i(t) &= i_f(t) + i_1(t) \\ &= i_f(t) + \sum_{i=1}^{\infty} a_i(t)R^i, \end{aligned} \tag{1}$$

where $i_f(t)$ is the current for $R = 0$. We note that the coefficients $a_n(t)$ satisfy[1]

$$\begin{aligned} a_1(t) &= -\int_0^t h(t-\sigma)i_f(t)\,d\sigma \\ a_n(t) &= -\int_0^t h(t-\sigma)a_{n-1}(\sigma)\,d\sigma, \\ &\qquad (n = 1, 2, \cdots) \end{aligned} \tag{2}$$

in which $h(t)$ is the current response to a unit impulse voltage. If we multiply the nth equation in (2) by R^n and then sum over all n, we find that $i_1(t)$ can be expressed as the solution of an integral equation,

$$\begin{aligned} i_1(t) &= \sum_{i=1}^{\infty} a_i(t)R^i \\ &= -Ra_1(t) \\ &\quad - R\int_0^t h(t-\sigma)i_1(\sigma)\,d\sigma. \end{aligned} \tag{3}$$

Let us denote the Laplace transform of any quantity $f(t)$ by the same symbol with an asterisk and an argument s. Then the relation in (3) yields

$$\begin{aligned} i_1^*(s) &= \int_0^{\infty} e^{-st}i_1(t)\,dt \\ &= -Rh^*(s)i_f^*(s) - Rh^*(s)i_1^*(s), \end{aligned} \tag{4}$$

using the convolution property. Hence we have

$$i_1(t) = -\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{Rh^*(s)i_f^*(s)}{1+Rh^*(s)}\,e^{st}\,ds, \tag{5}$$

for the perturbing current, where $h^*(s)$ is the current transfer function. If it is possible to find the inverse transform

$$\mathfrak{L}^{-1}\left\{\frac{Rh^*(s)}{1+Rh^*(\bar{s})}\right\} = \phi(R, t), \tag{6}$$

then $i_1(t)$ can be written

$$i_1(t) = -\int_0^t \phi(R, \sigma)i_f(t-\sigma)\,d\sigma. \tag{7}$$

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* Received by the PGCT, August, 1956.
[1] A. Papoulis, IRE Trans., vol. CT-3, p. 79; March, 1956.